Symbolic Descriptor and State Space Model of Interconnected Linear Systems with Algebraic Loops

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***Abstract*— This paper proposes a method which gives symbolic model of the interconnected linear system using the adjacency matrix which represent the system. When there exist any algebraic loops in the system, they are reduced to the self loops by the edge contraction. If the weight matrix of the self loop is nonsingular, it is removed and the symbolic state space model is obtained, otherwise the symbolic descriptor model is obtained. We introduce a new interactive graphical software tool which have been developed for modeling and simulation of control systems using the proposed method.**

I. Introduction

Computer software tools for handling symbolic models of the control system are useful for education and aids to get formulas involving the parameters which we want to vary [1], [2]. The symbolic models enable us to solve in a unified way a variety of control problems [3] and gives us the exact solutions, so there is no possibility of cumulative error due to rounding off. Although there are many tools which support numerical modeling in state space form of the linear systems [4], [5], [6], [7], there exists few tools which deal with symbolic ones [8], [7]. Those tools don't provide enough support for the symbolic modeling of the interconnected linear systems, such as MIMO systems, systems with algebraic loops, and systems which contain improper systems. Table I compares the features of some software tools which support the modeling in state space form of the interconnected linear system. ○ means the feature is available, △ means the feature is limited to SISO systems, and × means the feature is not available.

The descriptor form can represent differential equations of a dynamic system more effectively than the state-space form and preserve the independent physical parameters of the uncertainty structure [9]. And it can be used to describe the models of the improper linear systems as well as proper linear systems. A few tools supply the functions to obtain numerical descriptor form of the interconnected linear systems. There is no tools which can make the symbolic models in descriptor form of the interconnected linear systems as far as we know. Table II compares the features of some software tools which support the modeling in descriptor form of the interconnected linear system.

This paper proposes a method which gives symbolic model of the interconnected linear system using adjacency matrix which represent the system. When there is no algebraic loops in the system the method gives the symbolic model in state space form of the interconnected linear systems. When there exist any algebraic loops in the system, they are reduced to the self loops by the edge contraction. If the weight matrix of the self loop is nonsingular, it is removed and the symbolic state space model is obtained, otherwise the symbolic descriptor model is obtained. For the removal of the self loop the matrix inversion lemma is used recursively to calculate the inverse of matrix symbolically.

We introduce a new interactive graphical software tool Jamox (Java Agile MOdeling tool for Control Systems) which have been developed for modeling and simulation of control systems using the proposed method. Finally some illustrative examples are given.

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TABLE I

State Space Modeling of Interconnected System

	Matlab/ Simulink	Scilab/ Scicos	20-Sim	Maple Sim
Numerical	○	○	△	○
Symbolic	×	×	△	○
MIMO	○	○	×	○

TABLE II

Descriptor Form Modeling of Interconnected System

	Matlab/ Simulink	Scilab/ Scicos	20-Sim	Maple Sim
Numerical	○	○	×	×
Symbolic	×	×	×	×

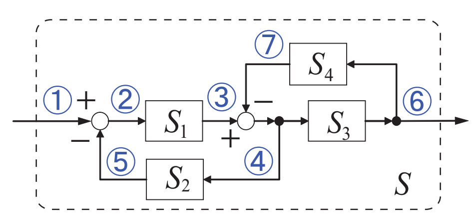

Fig. 1. Control System

II. Adjacency Matrix Representation

The adjacency matrix is a representation of a *graph* and the (i, j) element contains the adjacency information between the node i and the node j. When we regard an arrow of the block diagram as a node of graph, and a block and a summing point of the block diagram as an edge of graph, we can consider a causal block diagram as a weighted directed graph whose weights correspond to the values of the blocks. Therefore the data of a block digram can be represented as an adjacency matrix.

For example, the adjacency matrix of the system S shown in Fig.1 is given by

$$
W = \begin{array}{c}
\begin{array}{ccccccc} ① & ② & ③ & ④ & ⑤ & ⑥ & ⑦ \end{array} \\
\left[\begin{array}{c|c|c|c|c|c|c}
 & I & & & & & \\ \hline
 & & S_1 & & & & \\ \hline
 & & & I & & & \\ \hline
 & & & & S_2 & S_3 & \\ \hline
 & -I & & & & & \\ \hline
 & & & & & & S_4 \\ \hline
 & & & -I & & &
\end{array}\right]
\end{array}
\begin{array}{c} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \\ ⑦ \end{array}
\qquad (1)
$$

where S_i is an operator which represents input-output relationship of the system, I is an unit operator which transmits the input without modification, $-I$ is an unary minus operator whose output is opposite in sign to the input. Let v_i denotes the value of node i, then the following equation holds.

$$
\begin{bmatrix} 0 \\ v_2 \\ \vdots \\ v_7 \end{bmatrix} = W^T \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_7 \end{bmatrix}
$$

A. Adjacency Matrix of System in State Space Form

Consider a proper linear system described in state space form as

$$
qx = Ax + Bu \qquad (2)
$$
$$
y = Cx + Du \qquad (3)
$$

where q is differentiator for continuous-time systems and is time lead operator for discrete-time systems. And x is state variable, u is input variable, y is output variable and A, B, C, and D are constant matrices of appropriate dimensions.

Definition 1: We define the adjacency matrix representation of system (2)-(3) as

$$
\left[\begin{array}{c|c|c|c}
 & & B & D \\ \hline
 & & A & C \\ \hline
 & I/q & & \\ \hline
 & & &
\end{array}\right] \qquad (4)
$$

Then the equation

$$
\begin{bmatrix} 0 \\ x \\ q \cdot x \\ y \end{bmatrix} = \left[\begin{array}{c|c|c|c}
 & & B^T & D^T \\ \hline
 & & A^T & C^T \\ \hline
 & I/q & & \\ \hline
 & & &
\end{array}\right]^T \begin{bmatrix} u \\ x \\ q \cdot x \\ y \end{bmatrix}
$$

holds, where $\frac{I}{q}$ is a diagonal matrix with integrators as diagonal elements for continuous-time systems and is a diagonal matrix with time delay operators as diagonal elements for discrete-time systems. Figure 2 shows the block diagram of the system

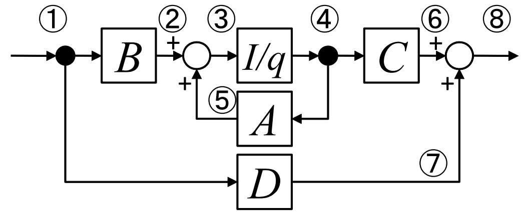

Fig. 2. Block Diagram of System in State Space Form

B. Adjacency Matrix of System in Descriptor Form

Consider a linear system described in descriptor system form as

$$
Eqx = Ax + Bu \qquad (5)
$$
$$
y = Cx + Du \qquad (6)
$$

where x is descriptor variable, u is input variable, y is output variable and E, A, B, C, and D are constant matrices of appropriate dimensions. If E is nonsingular, then the descriptor system form can be transformed into the state space form. Here we don't assume the nonsingularity of E. Add $(I - E)qx$ to both sides of (5), then the equation becomes as follows.

$$
\begin{aligned}
qx &= Ax + Bu + (I - E)qx \\
y &= Cx + Du
\end{aligned}
$$

Based on these equations we make the following definition.

Definition 2: We define the adjacency matrix representation of system (5)-(6) as

$$
\left[\begin{array}{c|c|c|c}
 & & B & D \\ \hline
 & & A & C \\ \hline
 & I/q & I{-}E & \\ \hline
 & & &
\end{array}\right] \qquad (7)
$$

Then the equation

$$
\begin{bmatrix} 0 \\ x \\ q \cdot x \\ y \end{bmatrix} = \left[\begin{array}{c|c|c|c}
 & & B^T & D^T \\ \hline
 & & A^T & C^T \\ \hline
 & I/q & (I{-}E)^T & \\ \hline
 & & &
\end{array}\right]^T \begin{bmatrix} u \\ x \\ q \cdot x \\ y \end{bmatrix}
$$

holds. Figure 3 shows the block diagram of the system Note that there exists an algebraic loop between node 3 and node 8.

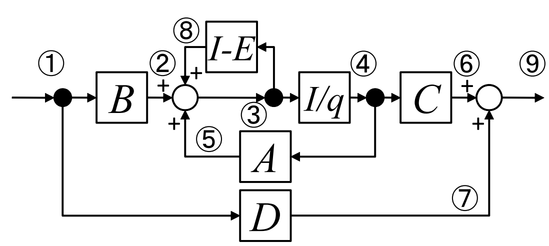

Fig. 3. Block Diagram of System in Descriptor Form

III. Interconnected System without Algebraic Loop

This section proposes a method to obtain the symbolic model of the interconnected linear system without algebraic loop. Suppose that the interconnected system contain proper linear system or constant and that it has no algebraic loop. In the first subsection we describe about contraction of series-edge and sorting of nodes which are used for simplification of adjacency matrix.

A. *Contraction of Series Edge and Sorting of Nodes*

The adjacency matrix of the system shown in Fig. 2 is given as

$$\begin{array}{cc} \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} & \textcircled{6} & \textcircled{7} & \textcircled{8} \end{matrix} & \\ \left[\begin{array}{c|c|c|c|c|c|c|c} & B & & & & & D & \\ \hline & & I & & & & & \\ \hline & & & I/q & & & & \\ \hline & & & & A & C & & \\ \hline & & I & & & & & \\ \hline & & & & & & & I \\ \hline & & & & & & & I \\ \hline & & & & & & & \end{array}\right] & \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \\ \textcircled{5} \\ \textcircled{6} \\ \textcircled{7} \\ \textcircled{8} \end{matrix} \end{array}$$

In this paper, we call a *series-edge* with a constant weight as a *constant series-edge*. It is possible to contract a constant series-edge to an edge whose weight is the product of the weights of the original series-edge. Contracting all constant series-edges of the graph corresponding to the system shown in Fig. 2 gives the following matrix.

$$\begin{array}{cc} \begin{matrix} \textcircled{1} & \textcircled{3} & \textcircled{4} & \textcircled{8} \end{matrix} & \\ \left[\begin{array}{c|c|c|c} & B & & D \\ \hline & & I/q & \\ \hline & A & & C \\ \hline & & & \end{array}\right] & \begin{matrix} \textcircled{1} \\ \textcircled{3} \\ \textcircled{4} \\ \textcircled{8} \end{matrix} \end{array}$$

Exchanging node-3 and node-4 (exchanging the 3rd row and the 4th row, and the 3rd column and the 4th column, respectively), leads to definition 1.

B. *Symbolic State Space Model of Interconnected System*

Consider an interconnected linear system with m external inputs and p external outputs. The following algorithm gives the adjacency matrix from which we can obtain the coefficients matrices of the system in state space form according to definition 1. In the algorithm we call each system contained in the interconnected system as subsystem.

Algorithm 1:

1) Get the adjacency matrix of the interconnected system using the adjacency matrices of the subsystems.
2) Contract all constant series-edges
3) Sort the nodes (exchange both rows and columns) such that the integrators (time delay operators) get lined up in diagonal in the left lower part of the adjacency matrix, where the first m columns and the last p rows are empty.
4) Sort the nodes which correspond to the external inputs and the external outputs by the number of those signals.

C. *Example*

Consider an interconnected linear system with two external inputs and two external outputs shown in Fig.4, where S_1 and S_2 are proper linear systems, and K_{11}, K_{12}, K_{21}, K_{22} are constant matrices of appropriate dimensions.

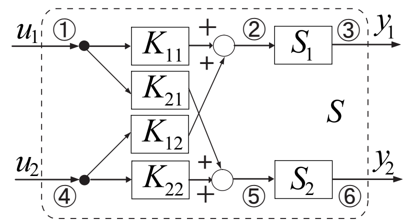

Fig. 4. Systems with Multiple External Inputs and Outputs

Contract all constant series-edges and sort the nodes (exchange both rows and columns) such that the integrators (time delay operators) get lined up in diagonal in the left lower part of the adjacency matrix, where the first two columns and the last two rows are empty, then we have

$$\begin{array}{cc} \begin{matrix} \textcircled{6} & \textcircled{1} & \textcircled{3} & \textcircled{8} & \textcircled{4} & \textcircled{9} & \textcircled{10} & \textcircled{5} \end{matrix} & \\ \left[\begin{array}{c|c|c|c|c|c|c|c} & & & & B_1K_{12} & B_2K_{22} & D_2K_{22} & D_1K_{12} \\ \hline & & & & B_1K_{11} & B_2K_{21} & D_2K_{21} & D_1K_{11} \\ \hline & & & & A_1 & & & C_1 \\ \hline & & & & & A_2 & C_2 & \\ \hline & & I_1/q & & & & & \\ \hline & & & I_2/q & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \end{array}\right] & \begin{matrix} \textcircled{6}(u_2) \\ \textcircled{1}(u_1) \\ \textcircled{3} \\ \textcircled{8} \\ \textcircled{4} \\ \textcircled{9} \\ \textcircled{10}(y_2) \\ \textcircled{5}(y_1) \end{matrix} \end{array}$$

Sorting the nodes which correspond to the external inputs and the external outputs by the number of the ports, we obtain

$$\tilde{W} = \begin{array}{cc} \begin{matrix} \textcircled{1} & \textcircled{6} & \textcircled{3} & \textcircled{8} & \textcircled{4} & \textcircled{9} & \textcircled{5} & \textcircled{10} \end{matrix} & \\ \left[\begin{array}{c|c|c|c|c|c|c|c} & & & & B_1K_{11} & B_2K_{21} & D_1K_{11} & D_2K_{21} \\ \hline & & & & B_1K_{12} & B_2K_{22} & D_1K_{12} & D_2K_{22} \\ \hline & & & & A_1 & & C_1 & \\ \hline & & & & & A_2 & & C_2 \\ \hline & & I_1/q & & & & & \\ \hline & & & I_2/q & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \end{array}\right] & \begin{matrix} \textcircled{1}(u_1) \\ \textcircled{6}(u_2) \\ \textcircled{3} \\ \textcircled{8} \\ \textcircled{4} \\ \textcircled{9} \\ \textcircled{5}(y_1) \\ \textcircled{10}(y_2) \end{matrix} \end{array}$$

Since matrix $\tilde{W}$ has the same structure as the adjacency matrix of a proper linear system defined in Eq. (4), the coefficient matrices of the system in state space form of the interconnected system are obtained as

$$A_S = \begin{bmatrix} \tilde{W}(3,5)^T & \tilde{W}(3,6)^T \\ \tilde{W}(4,5)^T & \tilde{W}(4,6)^T \end{bmatrix}^T = \begin{bmatrix} A_1 & \\ & A_2 \end{bmatrix}$$

$$B_S = \begin{bmatrix} \tilde{W}(1,5)^T & \tilde{W}(1,6)^T \\ \tilde{W}(2,5)^T & \tilde{W}(2,6)^T \end{bmatrix}^T = \begin{bmatrix} B_1K_{11} & B_1K_{12} \\ B_2K_{21} & B_2K_{22} \end{bmatrix}$$

$$C_S = \begin{bmatrix} \tilde{W}(3,7)^T & \tilde{W}(3,8)^T \\ \tilde{W}(4,7)^T & \tilde{W}(4,8)^T \end{bmatrix}^T = \begin{bmatrix} C_1 & \\ & C_2 \end{bmatrix}$$

$$D_S = \begin{bmatrix} \tilde{W}(1,7)^T & \tilde{W}(1,8)^T \\ \tilde{W}(2,7)^T & \tilde{W}(2,8)^T \end{bmatrix}^T = \begin{bmatrix} D_1K_{11} & D_1K_{12} \\ D_2K_{21} & D_2K_{22} \end{bmatrix}$$

IV. Interconnected System with Algebraic Loop

Since an algebraic loop is a loop of edges with constant weight, the contraction of series-edge makes it an self loop with constant weight. In this section we derive the condition that the self loop could be removed, i.e., the system which contains the self loop could be transformed into the equivalent system without self loop. And we divide the interconnected systems with algebraic loops into two cases: systems whose algebraic loops can be removed and systems whose algebraic loops can't be removed. The latter is further divided into two cases: one whose self loop is adjacent to the edge corresponding to the integrator (time delay operator), and the other whose self loop is not adjacent to the integrator (time delay operator) edge.

A. Condition for removal of algebraic loop

Figure 5 shows a portion of an interconnected system which makes an algebraic loop, where $K_i (i = 1, ..., n)$ are constant matrices of appropriate dimensions.

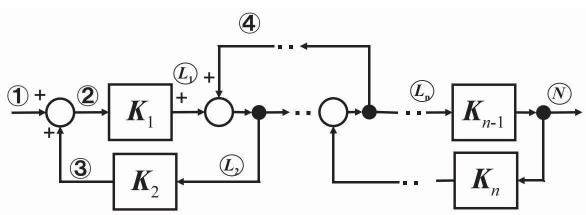

Fig. 5. Algebraic Loop

Contracting all constant series-edges which don't connect with the node which has any self-loops and sorting the nodes so that the nonzero diagonal elements in the adjacency matrix come together leads to the adjacency matrix

$$\begin{array}{cc} \begin{array}{ccc} v_{\overline{\ell}} & v_{\ell} & v_{\underline{\ell}} \end{array} & \\ \left[\begin{array}{c|c|c} & \overline{L} & * \\ \hline * & L & * \\ \hline * & \underline{L} & \end{array}\right] & \begin{array}{c} v_{\overline{\ell}} \\ v_{\ell} \\ v_{\underline{\ell}} \end{array} \end{array}$$

where v_ℓ contains the values of the nodes in the algebraic loop, $v_{\overline{\ell}}$ and $v_{\underline{\ell}}$ contain the values of the nodes outside the algebraic loop, and L has nonzero diagonal elements. Then we can obtain v_ℓ as follows.

$$v_\ell = \begin{bmatrix} \overline{L}^T \\ L^T \\ \underline{L}^T \end{bmatrix}^T \begin{bmatrix} v_{\overline{\ell}} \\ v_\ell \\ v_{\underline{\ell}} \end{bmatrix}$$

and

$$(I - L)v_\ell = \begin{bmatrix} \overline{L}^T \\ \underline{L}^T \end{bmatrix}^T \begin{bmatrix} v_{\overline{\ell}} \\ v_{\underline{\ell}} \end{bmatrix} \tag{8}$$

If $I - L$ is nonsingular, multiplying $(I - L)^{-1}$ to both sides of (8) from left and we have

$$v_\ell = \begin{bmatrix} ((I-L)^{-1}\overline{L})^T \\ ((I-L)^{-1}\underline{L})^T \end{bmatrix}^T \begin{bmatrix} v_{\overline{\ell}} \\ v_{\underline{\ell}} \end{bmatrix} \tag{9}$$

Then the adjacency matrix is given by

$$\begin{array}{cc} \begin{array}{ccc} v_{\overline{\ell}} & v_{\ell} & v_{\underline{\ell}} \end{array} & \\ \left[\begin{array}{c|c|c} & (I-L)^{-1}\overline{L} & * \\ \hline * & & * \\ \hline * & (I-L)^{-1}\underline{L} & \end{array}\right] & \begin{array}{c} v_{\overline{\ell}} \\ v_{\ell} \\ v_{\underline{\ell}} \end{array} \end{array}$$

Therefore if $I - L$ is nonsingular, where L is the weight of the self loop into which the algebraic loop is transformed by series-edge contraction, the self (algebraic) loop in the interconnected system can be removed.

B. State Space Model by Removing self loop

Suppose that the self (algebraic) loop in the interconnected system can be removed. The following algorithm gives the adjacency matrix from which we can obtain the coefficients matrices of the system in state space form according to definition 1.

Algorithm 2:
Replace (2) in algorithm 1 as follows.

a) Contract all constant series-edges which don't connect to the node with any self-loops.
b) If there exists any self-loops, i.e., there exists nonzero diagonal elements, collect up all nonzero diagonal elements to a diagonal submatrix by appropriately sorting the nodes. Let L denotes the submatrix. Multiply $(I - L)^{-1}$ to the elements in the the columns whose column numbers are equal to those of L from left and set O to L.
c) Go back to (a).

In algorithm 2, we have to obtain the inverse of the submatrix which contains the weights of the self loops. Though it is easy to calculate the inverse of matrix numerically, we use the matrix inversion lemma recursively to calculate the inverse of matrix symbolically to obtain the symbolic model.

C. Self loop with adjacency to Integrator

Suppose that the self (algebraic) loop in the interconnected system can not be removed and that the self loop is adjacent to the edge corresponding to the integrator (time delay operator). Contracting all constant series-edges which don't connect with the node which has any self-loops, removing all self loop which could be removed, and sorting the nodes (exchanging both rows and columns) so that the integrator (time delay operator) get lined up in diagonal in the left lower part of the adjacency matrix. leads to the adjacency matrix

$$
\begin{array}{cc}
\begin{array}{cccccc} u & x_1 & x_2 & \dot{x_1} & \dot{x_2} & y \end{array} & \\
\left[\begin{array}{c|c|c|c|c|c}
 & & & W_{14} & W_{15} & W_{16} \\ \hline
 & & & W_{24} & W_{25} & W_{26} \\ \hline
 & & & W_{34} & W_{35} & W_{36} \\ \hline
 & I_1/q & & & & \\ \hline
 & & I_2/q & & L & \\ \hline
 & & & & &
\end{array}\right] &
\begin{array}{c} u \\ x_1 \\ x_2 \\ \dot{x_1} \\ \dot{x_2} \\ y \end{array}
\end{array}
$$

where u is the value of input node, y is the value of output node, x_1, x_2, and $\dot{x}_1$ are the values of the nodes which don't connect to the self loop, and $\dot{x}_2$ is the value of nodes which are adjacent to the self loop. And I_1 and I_2 are unit matrix of appropriate dimensions. The self loop which connects with $\dot{x}_2$ can't be removed since $I_2 - L$ is singular by assumption, and is adjacent to the node corresponding to the integrator (time delay operator) I_2/q. Because this adjacent matrix has the same structure as the adjacency matrix of a descriptor system defined in (7), the coefficient matrices of the system in descriptor form of the interconnected system are obtained as

$$
\begin{aligned}
A &= \begin{bmatrix} W_{24} & W_{34} \\ W_{25} & W_{35} \end{bmatrix}, \qquad B = \begin{bmatrix} W_{14}^T & W_{15}^T \end{bmatrix}^T \\
C &= \begin{bmatrix} W_{26} & W_{36} \end{bmatrix}, \qquad D = \begin{bmatrix} W_{16} \end{bmatrix} \\
E &= \begin{bmatrix} I_1 & 0 \\ 0 & I_2 - L \end{bmatrix}
\end{aligned}
$$

The following algorithm gives the adjacency matrix from which we can obtain the coefficients matrices of the system in descriptor form according to definition 2.

Algorithm 3:

Replace (2) in algorithm 1 as follows.

a) Contract all constant series-edges which don't connect to the node which has any self-loops.
b) Remove all redundant nodes if present.
c) Remove all self loop if possible.
d) Go back to (a).

D. Self loop without adjacency to Integrator

Suppose that the self (algebraic) loop in the interconnected system can not be removed and that the self loop is not adjacent to the edge corresponding to the integrator (time delay operator). Contracting all constant series-edges which don't connect with the node which has any self-loops, removing all self loop which could be removed, and sorting the nodes (exchanging both rows and columns) so that the integrator (time delay operator) get lined up in diagonal in the left lower part of the adjacency matrix. leads to the adjacency matrix

$$
\begin{array}{cc}
\begin{array}{ccccc} u & x_1 & x_2 & \dot{x_1} & y \end{array} & \\
\left[\begin{array}{c|c|c|c|c}
 & & W_{13} & W_{14} & W_{15} \\ \hline
 & & W_{23} & W_{24} & W_{25} \\ \hline
 & & L & W_{34} & W_{35} \\ \hline
 & I_1/q & & & \\ \hline
 & & & &
\end{array}\right] &
\begin{array}{c} u \\ x_1 \\ x_2 \\ \dot{x_1} \\ y \end{array}
\end{array}
$$

where u is the value of input node, y is the value of output node, x_1 $\dot{x}_1$ are the values of the nodes which don't connect to the self loop, and x_2 is the value of nodes which are adjacent to the self loop. The self loop which connects with x_2 can't be removed since $I_2 - L$ is singular by assumption, and is not adjacent to the node corresponding to the integrator (time delay operator). And I_1 and I_2 are unit matrix of appropriate dimensions.

From the adjacency matrix we have the equation

$$x_2 = W_{23}x_1 + Lx_2 + W_{13}u$$

Adding $\dot{x}_2$ to both sides of the equation leads to

$$\dot{x}_2 = W_{23}x_1 + (L - I_2)x_2 + W_{13}u + \dot{x}_2$$

Using this equation and $x_2 = \frac{I_2}{q}\dot{x}_2$, we obtain

$$
\begin{array}{cc}
\begin{array}{cccccc} u & x_1 & x_2 & \dot{x_1} & \dot{x_2} & y \end{array} & \\
\left[\begin{array}{c|c|c|c|c|c}
 & & & W_{14} & W_{13} & W_{15} \\ \hline
 & & & W_{24} & W_{23} & W_{25} \\ \hline
 & & & W_{34} & L - I_2 & W_{35} \\ \hline
 & I_1/q & & & & \\ \hline
 & & I_2/q & & I_2 & \\ \hline
 & & & & &
\end{array}\right] &
\begin{array}{c} u \\ x_1 \\ x_2 \\ \dot{x_1} \\ \dot{x_2} \\ y \end{array}
\end{array}
$$

Because this adjacent matrix has the same structure as the adjacency matrix of a descriptor system defined in (7), the coefficient matrices of the system in descriptor form of the interconnected system are obtained as

$$
\begin{aligned}
A &= \begin{bmatrix} W_{24} & W_{34} \\ W_{23} & L - I_2 \end{bmatrix}, \qquad B = \begin{bmatrix} W_{14}^T & W_{13}^T \end{bmatrix}^T \\
C &= \begin{bmatrix} W_{25} & W_{35} \end{bmatrix}, \qquad D = \begin{bmatrix} W_{15} \end{bmatrix} \\
E &= \begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

The following algorithm gives the adjacency matrix from which we can obtain the coefficients matrices of the system in descriptor form according to definition 2.

Algorithm 4:

Replace (2) in algorithm 1 as follows.

a) Contract all constant series-edges which don't connect to the node which has any self-loops.
b) Remove all redundant nodes if present.
c) Remove all self loop if possible.
d) Add an integrator (time delay operator) as follows if there exists an self loop which can't be removed and go back to (a).
   i) Insert a column to the left of the column which has a self loop element.
   ii) Set an integrator (time delay operator) to the element which is adjacent to the self loop in the inserted column.
   iii) Subtract an unit matrix from the self loop element and set an unit matrix to the diagonal element in the same column.

V. Modeling and Simulation Tool

We have implemented the proposed method on Jamox which is a modeling and simulation tool for control systems. It enables us to obtain the symbolic model of the interconnected system from the specified input port to the output port in state space form, descriptor form, and transfer function form. Constant block, integrator block, time delay operator block, state space block, and transfer function block are available for making the block diagram of the interconnected system. The tag name, which is attached to each block to represent the block, is used to generate the symbolic model. We can set a number to the input port block, output port block, and block with state so that the elements of the input vector, the output vector, and the state vector are sorted in accordance with the number.

Some illustrative examples are given here. Figure 6 shows the block diagram of a hot strip mill looper system which has two inputs, two outputs, and four states. The symbolic model in state space form is shown in lower panel. Figure 7 shows the symbolic model of the interconnected system with some improper systems in descriptor form and transfer function form. Figure 8 shows the symbolic model of the mass spring damper system in descriptor form and transfer function form. The descriptor model preserves the structure of the independent physical parameters.

VI. Summary

This paper proposed a method which gives symbolic model of the interconnected linear system using the adjacency matrix which represent the system. When there exist any algebraic loops in the system, they are reduced to the self loops by the edge contraction. If the weight matrix of the self loop is nonsingular, it is removed and the symbolic state space model is obtained, otherwise the symbolic descriptor model is obtained. We have introduced a new interactive graphical software tool for modeling and simulation of control systems using the proposed method.

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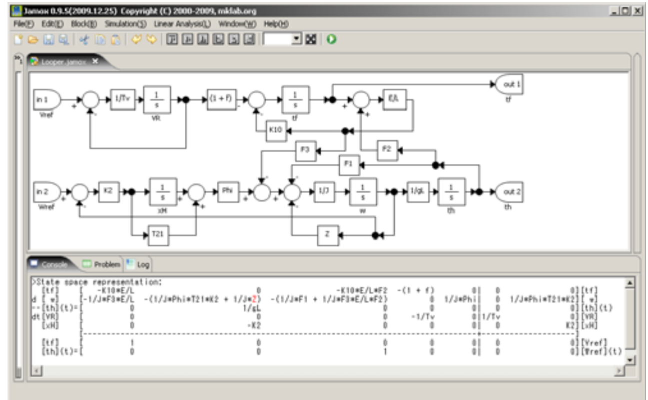

Fig. 6. Hot Strip Mill Looper System in Jamox

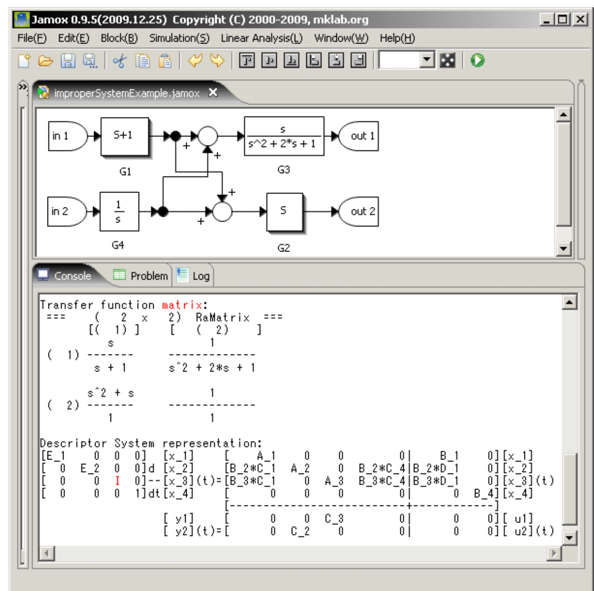

Fig. 7. Improper System in Jamox

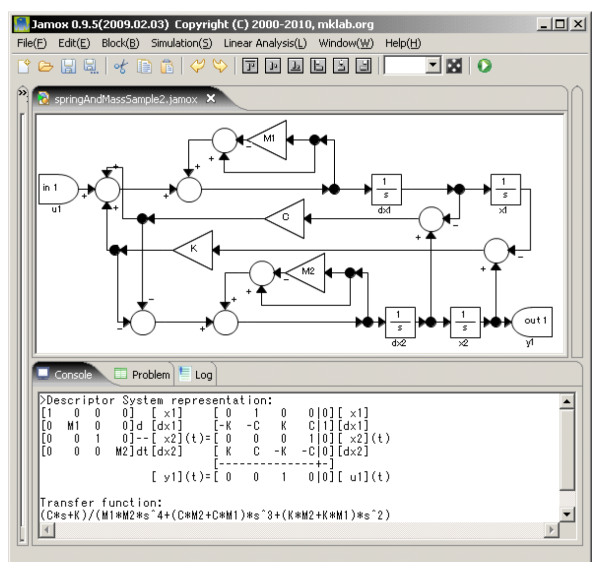

Fig. 8. Mass Sprint Damper System in Jamox